

Grade 6 Math Unit 7-Geometry

UNIT OVERVIEW

In grade 6, instruction time should focus on four critical areas. This unit does not directly relate to one critical focus area. This unit focuses on additional content for development. (See Connections for explanation)

Students in Grade 6 build on their work with area in elementary grades

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 6

Domain 6.G Geometry

Cluster Statement Solve real-world and mathematical problems involving area, surface area, and volume.

Standard 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Standard 6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Standard 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Standard 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

CONTENT ELABORATIONS

Students will build upon prior work in **6.NS.8**

Students continue to understand that area is the number of squares needed to cover a plane figure. Find the area of triangles is introduced in relationship to the area of rectangles - a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is $\frac{1}{2}$ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is $\frac{1}{2}bh$ or $(b \times h) / 2$. Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles. Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together.

6.G.1

Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding why the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for all students.

Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi and kites. Students can use tools such as the Isometric Drawing Tool on NCTM's Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D.

Examples:

- Find the area of a triangle with a base length of three units and a height of four units.
- A rectangle measures 3in. by 4in. If the lengths of each side double, what is the effect on the area?
- The area of the rectangular school garden is 24 square units. the length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?

MP.1-MP.8 should be emphasized.

6.G.2

Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The unit cube was $1 \times 1 \times 1$. In 6th grade, the unit cube will have fractional edge lengths (i.e., $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$). Students find the volume of the right rectangular prism with these unit cubes. For example, a right rectangular prism has edges of $1\frac{1}{4}$ ", 1" and $1\frac{1}{2}$ ". The volume can be found by recognizing that the unit cube would be $\frac{1}{4}$ " on all edges, changing the dimensions to $\frac{5}{4}$ ", $\frac{4}{4}$ ", and $\frac{6}{4}$ ". The volume is the number of unit cubes making up the prism ($5 \times 4 \times 6$), which is 120 unit cubes each with a volume of $\frac{1}{64}$ ($\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$). This can also be expressed as $\frac{5}{4} \times \frac{6}{4} \times \frac{4}{4}$ or $\frac{120}{64}$. Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations. In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.
MP.1 - MP.8 should be emphasized.

6.G.3

Students are given the coordinates of polygons to draw in the coordinate plane. If both x-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the y-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, students solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane.
Example: On a map, the library is located at (-2, 2), the city hall building is located at (0, 2) and the high school is located at (0, 0). Represent the locations as points on a coordinate grid with a unit of 1 mile.
- What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
- What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?
MP.1 - MP.8 should be emphasized.

6.G.4

A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure. It is very important for students to physically manipulate materials and make connections to the symbolic and more abstract aspects of geometry. Students construct models and nets of three-dimensional figures, and describe them by the number of edges, vertices and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.
Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations. Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.
Examples :
- Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?
- Create the net for a given prism or pyramid, and then use the net to calculate the surface area.
MP.1 - MP.7 should be emphasized.

UNIT VOCABULARY

base
height
composite figure
parallelogram
congruent
polygon
formula

rhombus
slant height
cubic units
surface area
lateral face
three-dimensional figure

prism
triangular prism
pyramid
vertex
rectangular prism
volume

BIG IDEAS

ENDURING UNDERSTANDINGS

- The area of irregular and regular polygons can be found by decomposing the polygon into rectangles and triangles.
- Manipulatives and the construction of nets may be used in computing the surface area of rectangular and triangular prisms, and volume of right rectangular prism.
- Formulas may be used to compute the areas of polygons and volumes of right rectangular prisms.
- Appropriate units of measure should be used when computing the area (square units) of polygons, and surface area (square units) and volume of prisms (cubic units).
- Views of rectangular and triangular prisms may be interpreted and sketched to provide a 2- dimensional representation(nets) of a three dimensional figure.
- Dimensions of solid figures may have fractional lengths.
- The volume of a solid figure is the number of same sized cubes filling the space so that there are no gaps and overlaps.

ESSENTIALS QUESTIONS

Choose a few questions based on the needs of your students

- How can we find the area of figures?
- How can we cut and rearrange irregular polygons in order to find their area?
- How can we use one figure to determine the area of another?
- How do we figure the area of a shape without a formula for that shape?
- How are the areas of geometric figures related to each other?
- How can I use manipulatives and nets to help compute the surface areas of rectangular and triangular prisms?
- What kinds of problems can be solved using surface areas of rectangular and triangular prisms?
- How can I interpret and sketch views of rectangular and triangular prisms?
- How can you model finding surface area and volume of rectangular and triangular prisms?
- How can I use formulas to determine the volume of right rectangular prisms?
- How can I determine the appropriate units of measure that should be used when computing the volumes of a right rectangular prism?
- What kinds of problems can be solved using volumes of fundamental solid figures?
- How does the fractional edge length affect the volume of a prism?

CONNECTIONS

Students in Grade 6 build on their work with area in elementary grades by reasoning about relationships among shapes to determine area, surface area and volume.

An understanding of how to find the area, surface area and volume of an object is developed in Grade 5 and should be built upon in Grade 6 to facilitate understanding of the formulas found in Measurement and Data and when to use the appropriate formula.

The use of floor plans and composite shapes on dot paper is a foundational concept for scale drawing and determining the actual area based on a scale drawing in Grade 7.

Standards for Mathematical Practice (SMP)

- MP.1** Make sense of problems and persevere in solving them
- MP.2** Reason abstractly and quantitatively
- MP.3** Construct viable arguments and critique the reasoning of others
- MP.4** Model with mathematics
- MP.5** Use appropriate tools strategically
- MP.6** Attend to precision
- MP.7** Look for and make use of structure (Deductive reasoning)
- MP.8** Look for and express regularity in repeated reasoning (Inductive Reasoning)

SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS INVOLVING AREA, SURFACE AREA, AND VOLUME

CONTENT

SKILLS

A.	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. 1. Recognize and know how to compose and decompose polygons into triangles and rectangles. 2. Compare the area of a triangle to the area of the composed rectangle. 3. Apply the techniques of composing and/or decomposing to find the area of triangles, special quadrilaterals and polygons to solve mathematical and real world problems. 4. Discuss, develop and justify formulas for triangles and parallelograms (6th grade introduction)
B.	Find the volume of a right rectangular prism with fractional edge lengths.	Find the volume of a right rectangular prism with fractional edge lengths. 1. Model the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. 2. Know how to calculate the volume of a right rectangular prism. 3. Apply volume formulas for right rectangular prisms to solve real-world and mathematical problems involving rectangular prisms with fractional edge lengths.
C.	Given coordinates, draw polygons in the coordinate plane and use the coordinates to find the length of sides of the polygon.	Given coordinates, draw polygons in the coordinate plane and use the coordinates to find the length of sides of the polygon. 1. Draw polygons in the coordinate plane. 2. Use coordinates (with the same x- coordinate or the same y- coordinate) to find the length of a side of a polygon. 3. Apply the technique of using coordinates to find the length of a side of a polygon drawn in the coordinate plane to solve real-world and mathematical problems.
D.	Use nets to find surface area of three-dimensional figures made of rectangles and triangles.	Use nets to find surface area of three-dimensional figures made of rectangles and triangles. 1. Know that 3-D figures can be represented by nets. 2. Represent three-dimensional figures using nets made up of rectangles and triangles. 3. Apply knowledge of calculating the area of rectangles and triangles to a net, and combine the areas for each shape into one answer representing the surface area of a 3-dimensional figure.
<p>Common Core Model Curriculum McGraw-Hill, Glencoe Math, Chapters 9-10 Georgia Math framework, Grade 6 Unit5 Manipulatives Smart Board resources Hands-On Standards Cubes Tools on NCTMs Illuminations Dynamic Paper Tool on NCTM Illuminations</p>		